Solutions

Math 2D Quiz 5 Morning - February 18th

Please put name and ID on *both* sides for grading and redistribution!

Show all of your work. *There is a question on the back side.

1. Consider the surface equation $3x + 2y + z = e^{xyz}$. Let $P = (0, 0, 1)$.

(a) Find the equation of the tangent plane at P.

(b) Find the symmetric equation of the normal line to the surface at P.

(c) Find (using Implicit Differentiation / Implicit Function Theorem):

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}, \quad \text{and (be careful with this one)} \quad \frac{\partial x}{\partial y}.$$

Your answers for (c) are functions - I do NOT want you to evaluate them at the point P.

a) First $F(x,y,z) = 3x + 2y + z - e^{xyz} = 0$ is our level surface.

$\nabla F(x,y,z) = \langle 3 - yze^{xyz},\ 2 - xze^{xyz},\ 1 - xye^{xyz} \rangle$ +1

so $\nabla F(0,0,1) = \langle 3, 2, 1 \rangle$ at $P = (0,0,1)$. // Is the normal vector for plane.

Thus, the eqn. of the tangent plane is $\boxed{3(x) + 2(y) + (z-1) = 0}$ +1

b) From (a), it is $\boxed{\dfrac{x}{3} = \dfrac{y}{2} = z - 1}$ +1

c)

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = -\frac{3 - yze^{xyz}}{1 - xye^{xyz}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{2 - xze^{xyz}}{1 - xye^{xyz}}$$

$$\frac{\partial x}{\partial y} = -\frac{F_y}{F_x} = -\frac{2 - xze^{xyz}}{3 - yze^{xyz}}$$

Re-used F_x, F_y, F_z from part (a).

+2

2. Let $f(x, y) = e^{xy} \sin(x^2 + y^2)$.
(a) Compute $\nabla f(x, y)$.
(b) Compute $D_{\mathbf{u}} f(\sqrt{\pi}, 0)$ in the direction of $\mathbf{u} =< 1, -\sqrt{3} >$. ($\hat{\mathbf{u}} =< \frac{1}{2}, -\frac{\sqrt{3}}{2} >$).
(c) This function was originally written as

$$f(r, \theta) = e^r \sin(\theta), \quad r = st, \quad \theta = s^2 + t^2.$$

Use the chain rule to compute

$$\frac{\partial f}{\partial s} \quad \text{and} \quad \frac{\partial f}{\partial t}$$

without plugging in for r, θ as functions of s, t (like we did in section).

a) $\nabla f(x,y) = < f_x(x,y) \, , \, f_y(x,y) >$

+1 +1

$= < ye^{xy}\sin(x^2+y^2) + 2xe^{xy}\cos(x^2+y^2); \; xe^{xy}\sin(x^2+y^2) + 2ye^{xy}\cos(x^2+y^2) >$

b) First, $\nabla f(\sqrt{\pi}, 0) = < 0 + 2\sqrt{\pi}\cos(\pi) \, , \, 0 + 0 >$

$= < -2\sqrt{\pi} \, , \, 0 > .$

$\hat{u}$

Thus, $D_u f(\sqrt{\pi}, 0) = < -2\sqrt{\pi}, 0 > \cdot < \frac{1}{2}, -\frac{\sqrt{3}}{2} >$

$= \boxed{-\sqrt{\pi}}$ +1

c) $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial s} + \frac{\partial f}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} = \boxed{e^r \sin\theta \cdot t + e^r \cos\theta \cdot 2s}.$ +1

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial t} + \frac{\partial f}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} = \boxed{e^r \sin\theta \cdot s + e^r \cos\theta \cdot 2t}$ +1